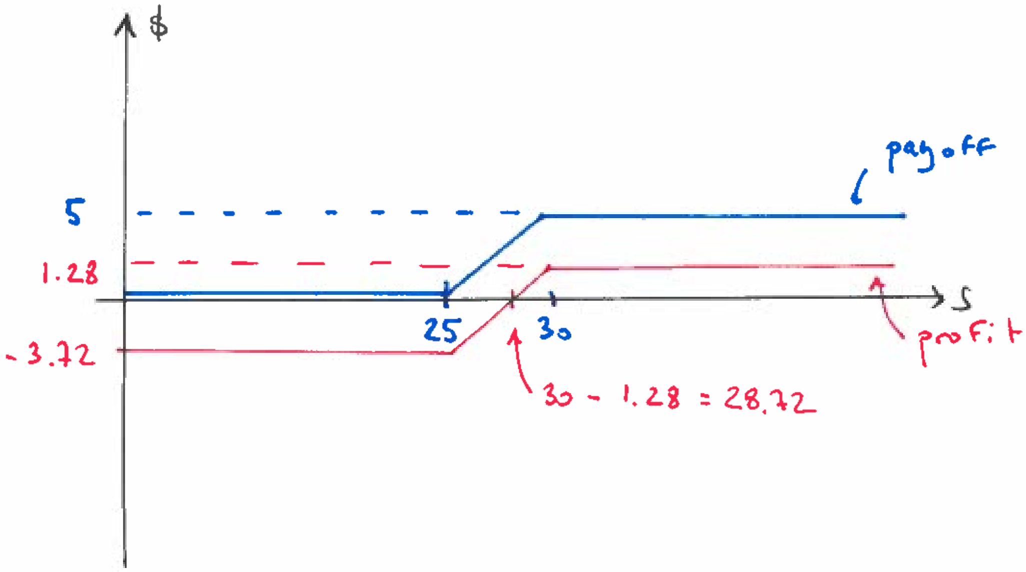
Problem Set 3

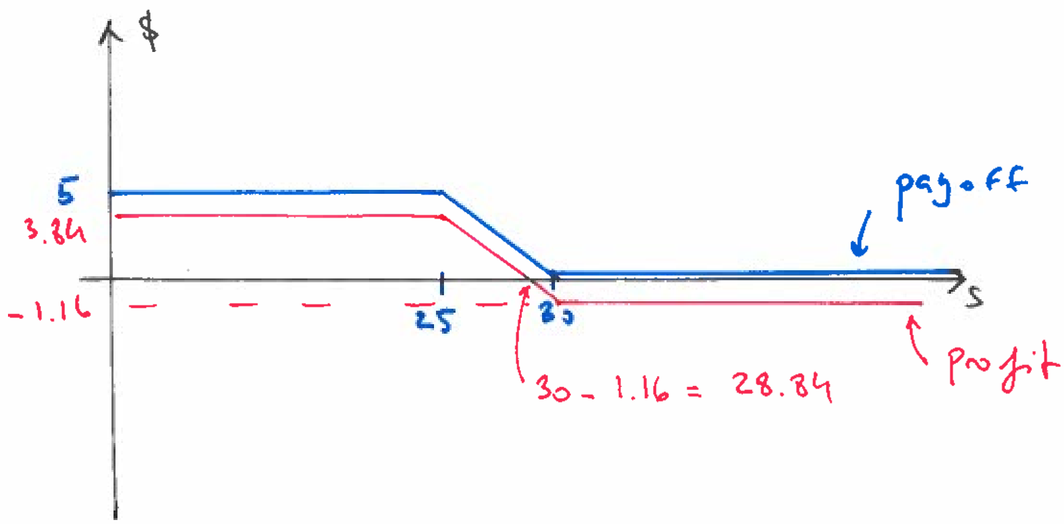
① We compute first the price of the put options using put-call parity.

Strike	25	30	35
6M	0.28	1.44	3.99
12M	0.70	2.14	4.57

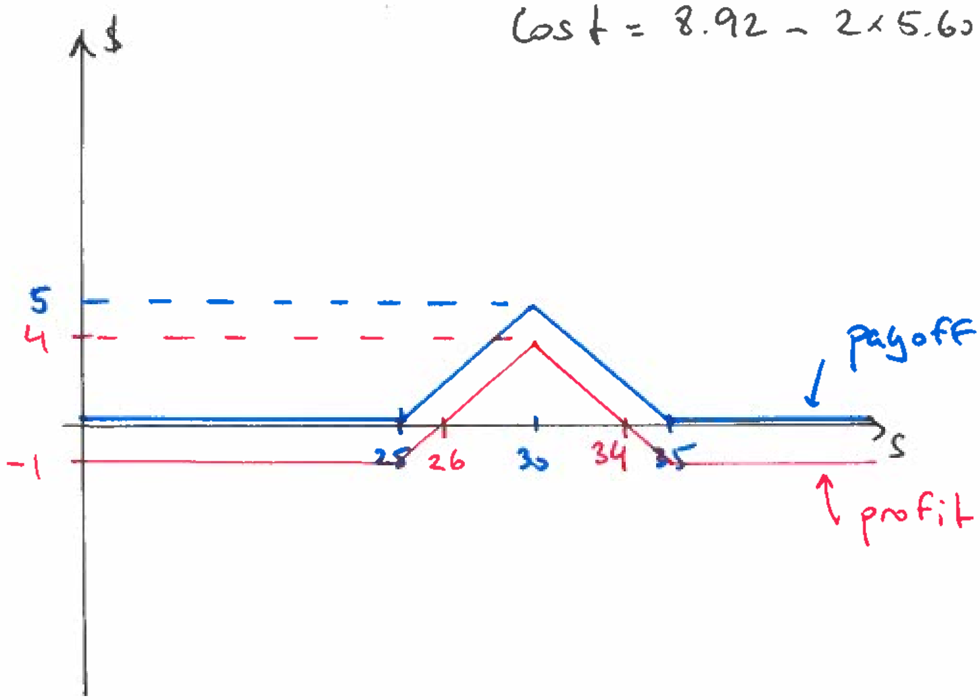
a. Cost = 7.90 − 4.18 = \$ 3.72



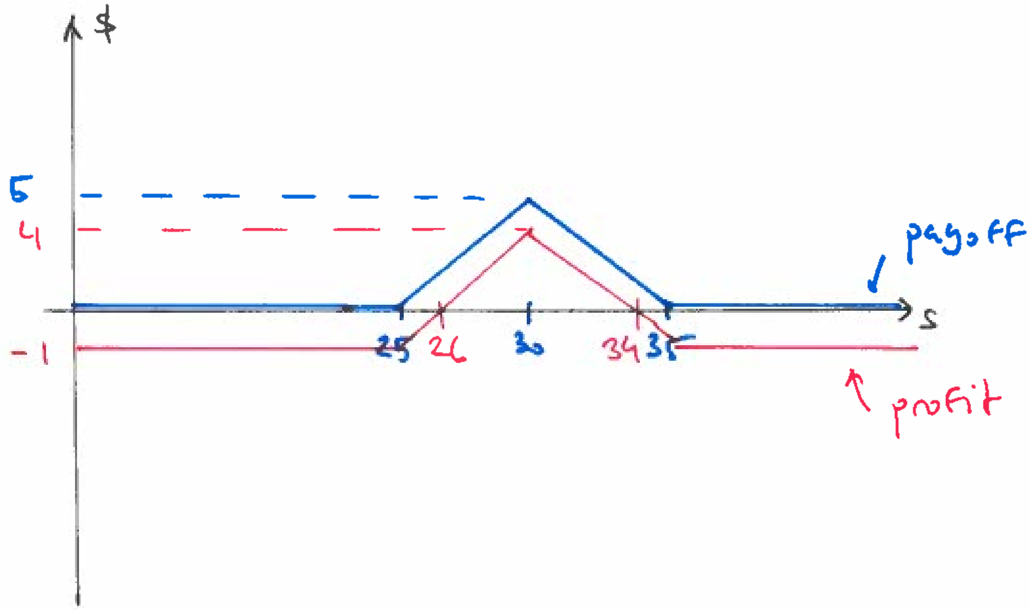
b. Cost = 1.44 − 0.28 = \$ 1.16



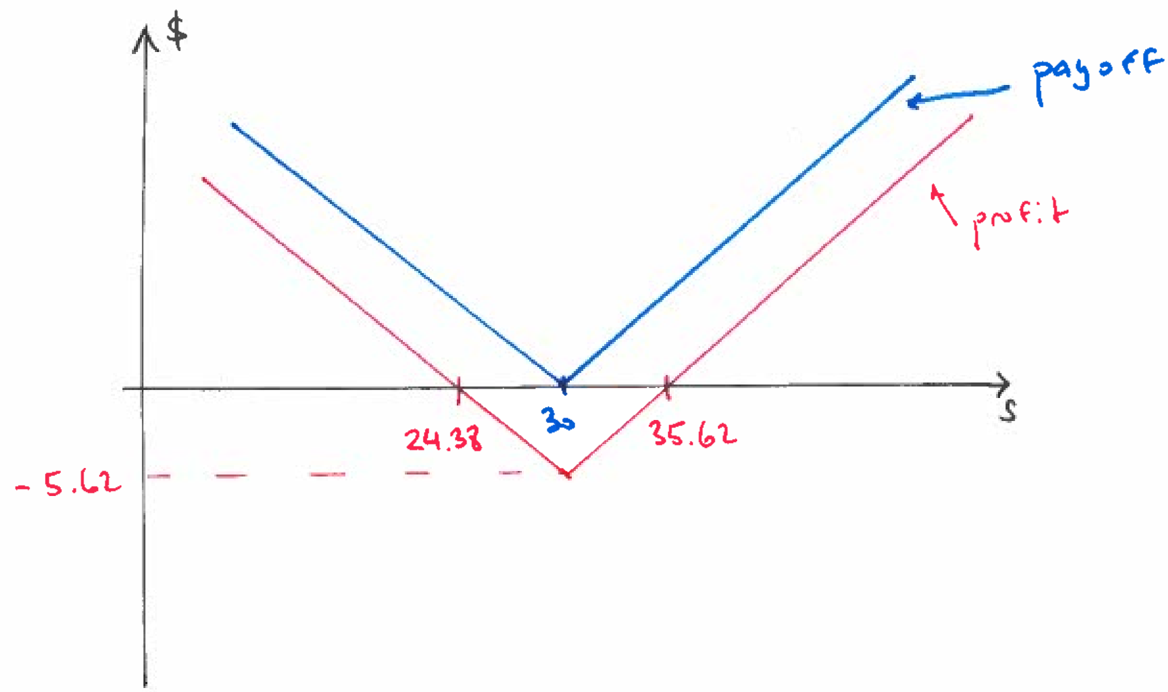
c. Cost = 8.92 − 2×5.60 + 3.28 = \$1.00



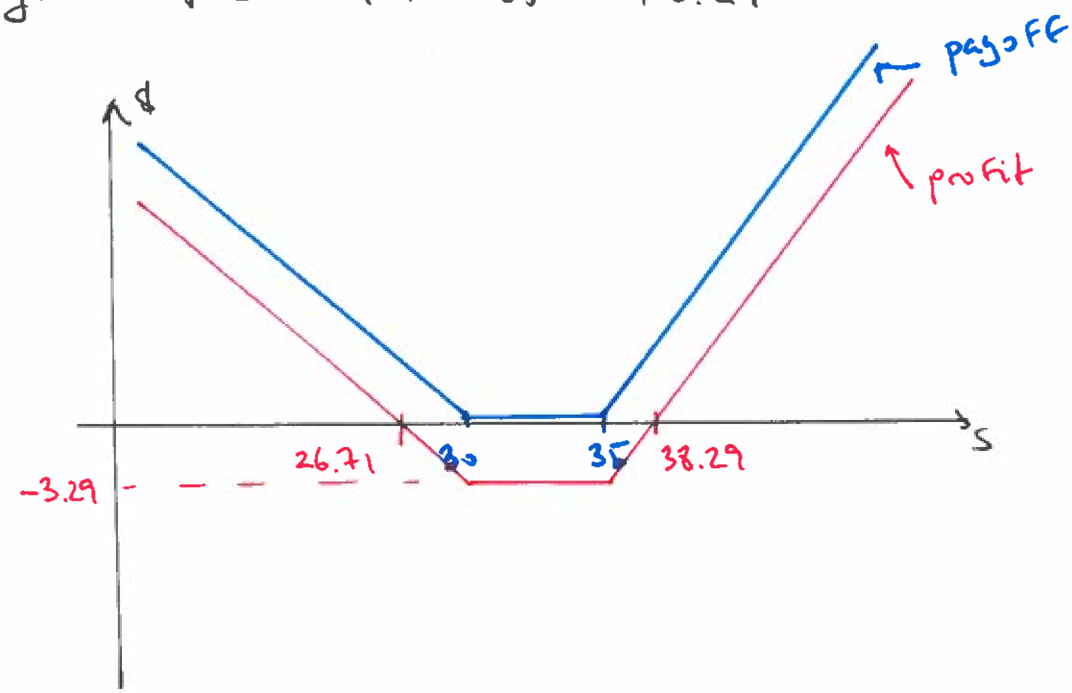
d. Cost = 0.70 − 2×2.14 + 4.57 = \$1.00



e. Cost = 1.44 + 4.18 = \$5.62



f. Cost = 1.44 + 1.85 = \$3.29



(2) The lower bound for the put is

$$P \geq \max\left(100e^{-0.06 \times 4/12} - 96, 0\right) = 2.02$$

Therefore, the price of the put is too cheap. We can try to make a negative price call since

$$C = P + S - \underbrace{Ke^{-rT}}$$

$$= 1.80 + 96 - 98.02 = -0.22$$

In terms of cash flows

	$t=0$	$t = 4/12$: $S_t \leq 100$	$S_t > 100$
Buy Put	-1.80	$100 - S_t$	0
Buy Stock	-96	S_t	S_t
Borrow	$+98.02$	-100	-100
Total	$+0.22$	0	$S_t - 100 > 0$.

We have created an instrument with the payoff of a call but negative cost, i.e., generating a positive cash flow today.

(3) The lower bound for the call is

$$c \geq \max\left(104 - 100\, e^{-0.06 \times 6/12}, 0\right) = 6.96$$

The call is too cheap!

Since

$$P = c - S + Ke^{-rT}$$
$$= 6.70 - 104 + 97.04 = -0.26$$

we can create a negative price instrument that pays like a put.

	$t=0$	$t = 6/12$: $S_t \leq 100$	$t = 6/12$: $S_t > 100$
Buy call	-6.70	0	$S_t - 100$
Sell stock	$+104$	$-S_t$	$-S_t$
Deposit	-97.04	$+100$	$+100$
Total	$+0.26$	$100 - S_t > 0$	0

(4) According to put-call parity, the call should cost

$$C = 8 + 200 - 200e^{-0.05 \times 9/12} = 15.36 > 15$$

Therefore, we should buy the call and sell the "synthetic" call to generate $15.36 - 15 = \$0.36$. More specifically, the cash flows and operations are

	$t=0$	$t=9/12$: $S_t \leq 200$	$t=9/12$: $S_t > 200$
Buy call	-15	0	$S_t - 200$
Sell put	$+8$	$-(200 - S_t)$	0
Sell stock	$+200$	$-S_t$	$-S_t$
Deposit	-192.64	200	200
Total	$+0.36$	0	0

It's an arbitrage

(5) Put-call parity says

$$c = 13 + 100 - 100\, e^{-0.06 \times 6/12} = 15.96 > 15$$

Again, the call is too cheap. Therefore, buy the call, sell the put, sell the stock and make a deposit of 97.04 for six months at 6% per year.

This generates an arbitrage profit of \$ 0.96 per share.

(6) The price of the put is its lower bound, that is

$$P = \max\left(280\, e^{-0.05} - 250, 0\right) = \$16.34.$$

(7) Similarly, the price of the call is its lower bound

$$c = \max\left(160 - 150\, e^{-0.10 \times 0.5}, 0\right) = \$17.36$$

(3)

a. False. The time-value of an American call option is always positive in this case. Early exercise distroys value.

b. True. If the option is deep-in-the-money, its unlikely the the option will become OTM, therefore makes sense to exercise, collect the strike minus the stock and invest at the risk-free rate.

c. False. In this case the time value of a European call is always positive (see graph in the slides).

d. True. The time value might be negative for stock prices low enough since the lower bound for the European put is below intrinsic value in that range of prices.